

BASIC LAPLACE TRANSFORMATION PROPERTIES

| PROPERTIES | TIME DOMAIN | FREQUENCY DOMAIN |
|-----------------------|--|--|
| Independent Variable | t | s |
| Signal Representation | $f(t)$ | $F(s)$ |
| Uniqueness | $\mathcal{L}^{-1}\{F(s)\} (=) [f(t)]u(t)$ | $\mathcal{L}\{f(t)\} = F(s)$ |
| Linearity | $Af_1(t) + Bf_2(t)$ | $AF_1(s) + BF_2(s)$ |
| Integration | $\int_0^t f(\tau) d\tau$ | $\frac{F(s)}{s}$ |
| Differentiation | $\frac{df(t)}{dt}$ $\frac{d^2f(t)}{dt^2}$ $\frac{d^3f(t)}{dt^3}$ | $sF(s) - f(0^-)$ $s^2F(s) - sf(0^-) - f'(0^-)$ $s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$ |
| t-Translation | $[f(t-a)]u(t-a)$ | $e^{-\alpha t} F(s)$ |
| s-Translation | $e^{-\alpha t} f(t)$ | $F(s + \alpha)$ |
| Scaling | $f(at)$ | $\frac{1}{a} F\left(\frac{s}{a}\right)$ |
| Final Value | $\lim_{t \rightarrow \infty} f(at)$ | $\lim_{s \rightarrow 0} s F(s)$ |
| Initial Value | $\lim_{t \rightarrow 0^+} f(at)$ | $\lim_{s \rightarrow \infty} s F(s)$ |